M.Sc. Sem II 9th Paper

TOPOLOGY

Normal space

Let X be a topological space. Then X is called a normal space if for each pair of disjoint closed subsets F_1 and F_2 of X, there exists a pair of disjoint open sets G and H such that

$$F_1 \subseteq G, \quad F_2 \subseteq H.$$

T_4- space : A normal T_1-space is called T_4- space.

Q. (a) Prove that every normal space is not regular space.

(b) Prove that every normal space is not a T_1- space.

Soln We shall prove by showing an example.

Let $X = \{a, b, c\}$.

Let $T = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$

T is a topology on X. Let us verify it.

~~closed~~

(a) $\phi \in T$, $X \in T$

(b) $\phi \cup X = X \in T$, $\phi \cup \{b\} = \{b\} \in T$

$\{b\} \cup \{c\} = \{b,c\} \in T$, $\{b\} \cup X = X \in T$

Similarly others.

(c) $\phi \cap X = \phi \in T$,

Intersection of all members with $\phi = \phi$ and $\phi \in T$.

Also, $X \cap \{b\} = \{b\} \in T$,

$X \cap \{c\} = \{c\} \in T$

$X \cap \{b,c\} = \{b,c\} \in T$

Similarly others. So, T is a topology on X.

closed sets of $(X,T) = X, \phi, \{a,c\}, \{a,b\}, \{a\}$.

If F_1 and F_2 are disjoint closed subsets of X then either of F_1 or $F_2 = \phi$. Let $F_1 = \phi$.

Then ϕ and X are also disjoint open sets such that $F_1 \subseteq \phi$, $F_2 \subseteq X$. $\Rightarrow$ (X,T) is a normal space.

(X,T) is not a regular space because $c \notin \{a\}$ and the only open superset of closed set $\{a\}$ is X which also contains c.

(X,T) is not a T_1-space since the singleton set $\{b\}$ is not a closed set.